Deterministic chaos (or why we care about initial conditions), and model inadequacy (or why this makes data assimilation harder)

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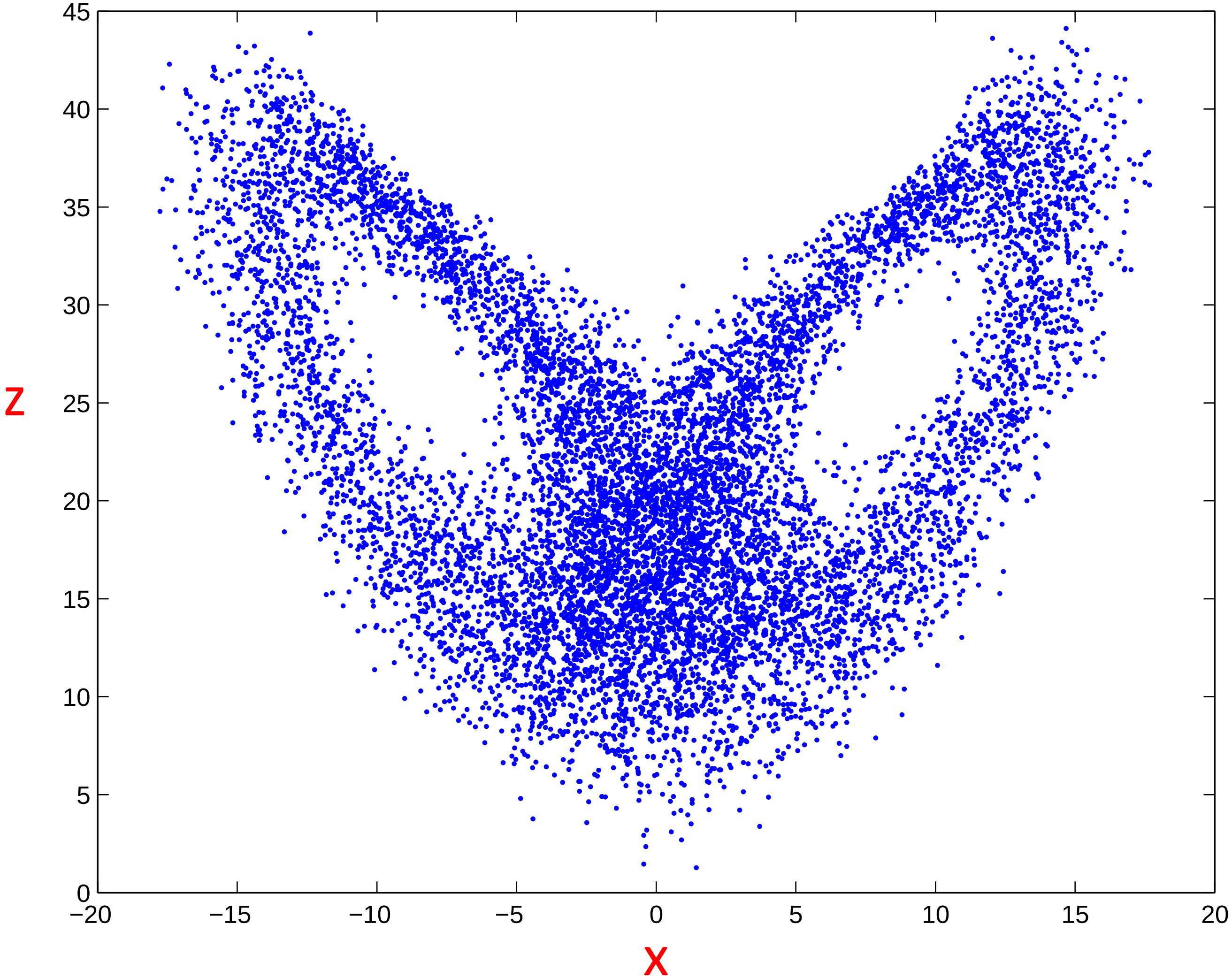
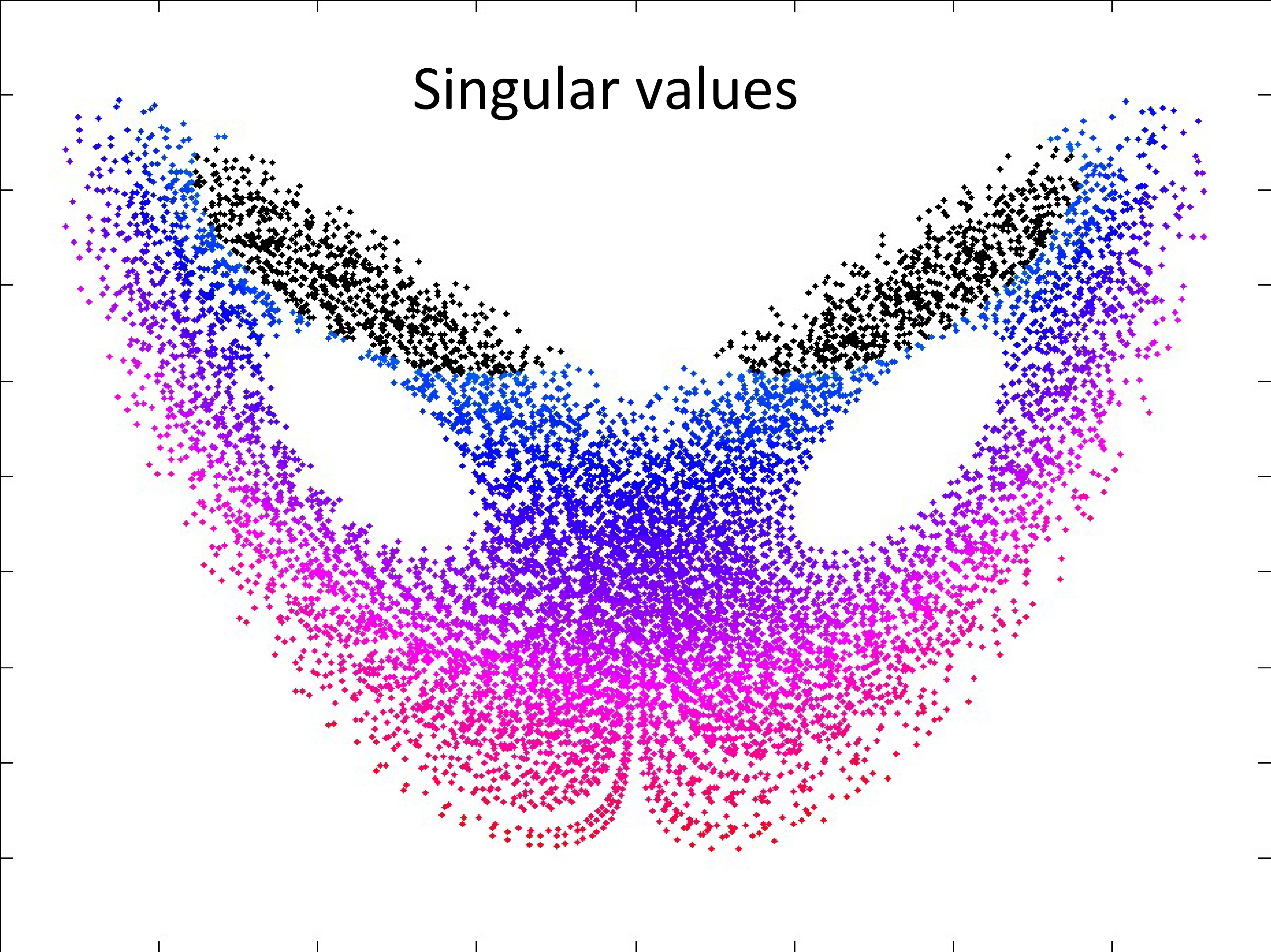
Attractor: Lorenz ‘63



Attractors: Why care?

- Attractors reflect the distribution of states realizable by a system.
  - Attractors define a system’s climatology
  - Attractors define a system’s “balance”
  - Attractors provide a basis for ensemble construction
- We don’t know if the atmosphere has an attractor, but NWP models almost certainly do.

Singular values

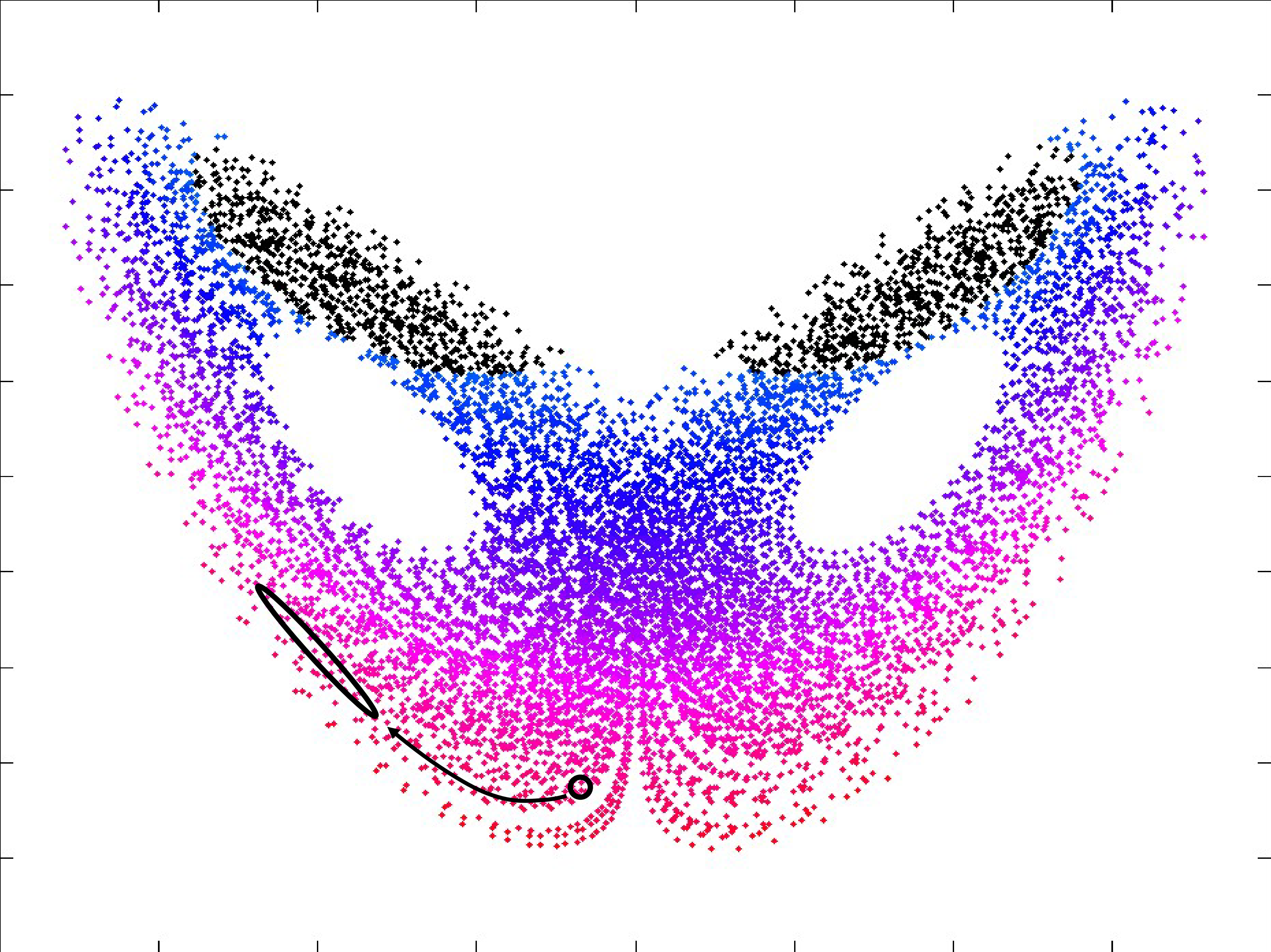


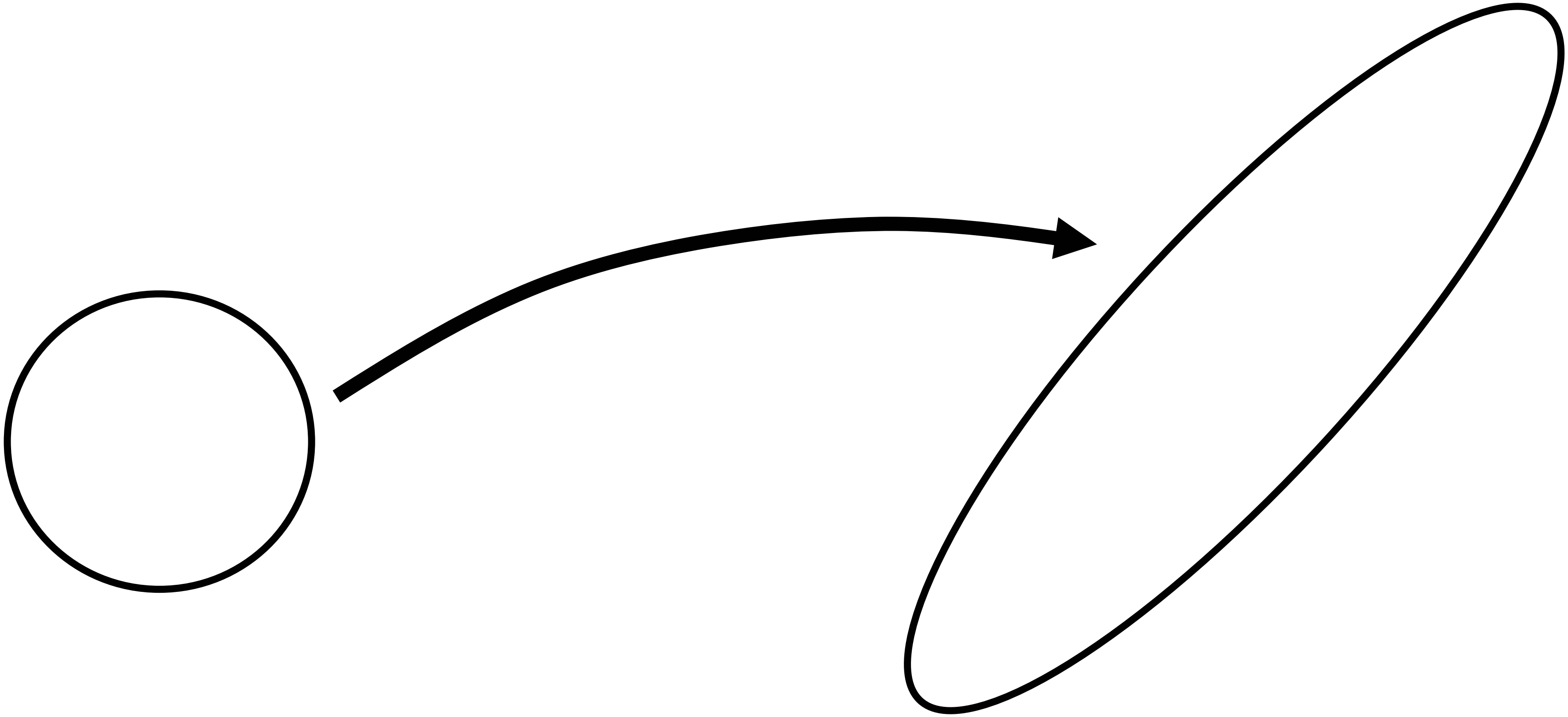
Singular values

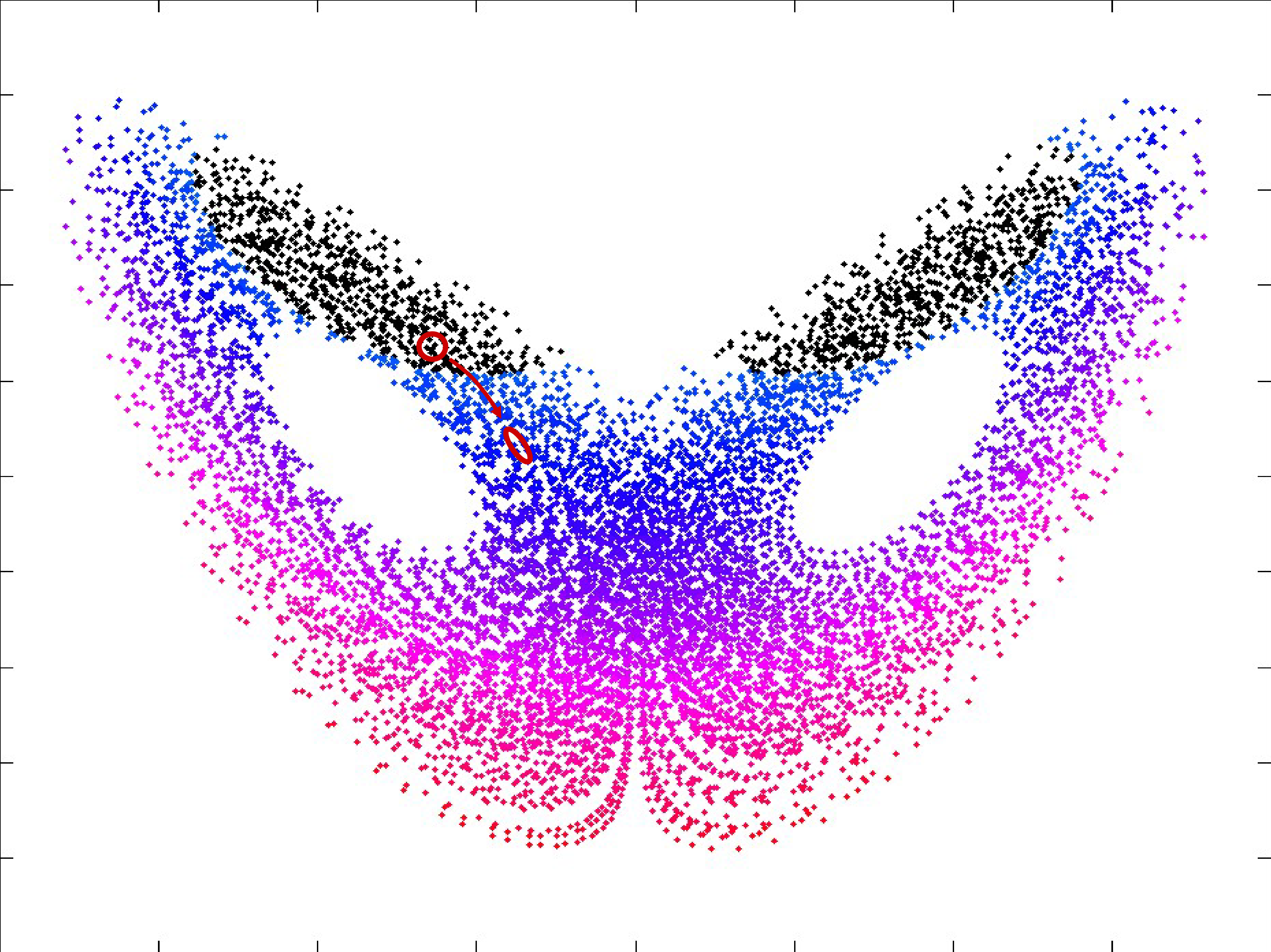
- Indicate the factor by which initial error will grow for infinitesimal errors over a finite time at a particular location (singular vectors give the directions).

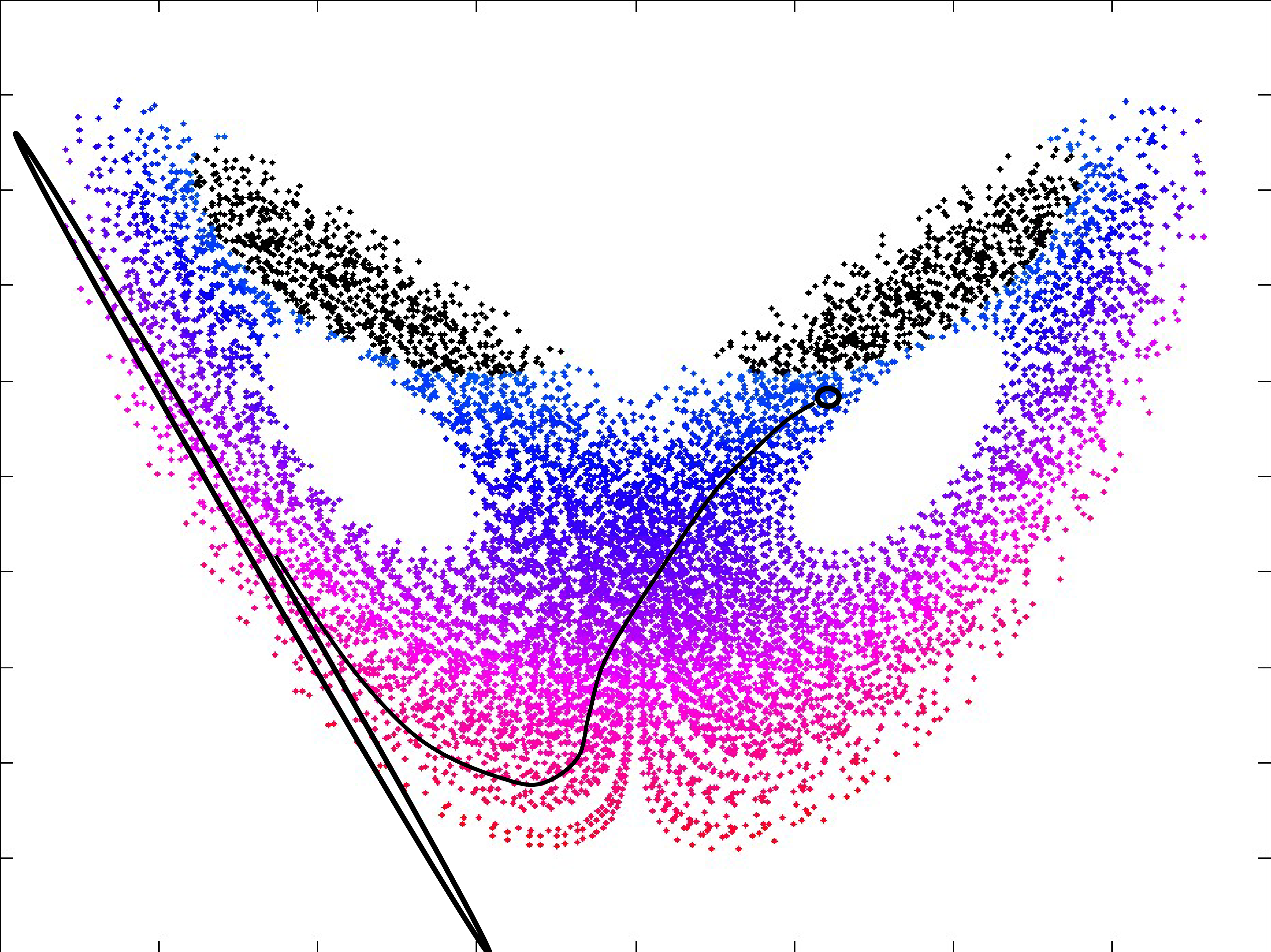
$$\left|\boldsymbol{\varepsilon}(t)\right| = \sigma \left|\boldsymbol{\varepsilon}(0)\right|$$

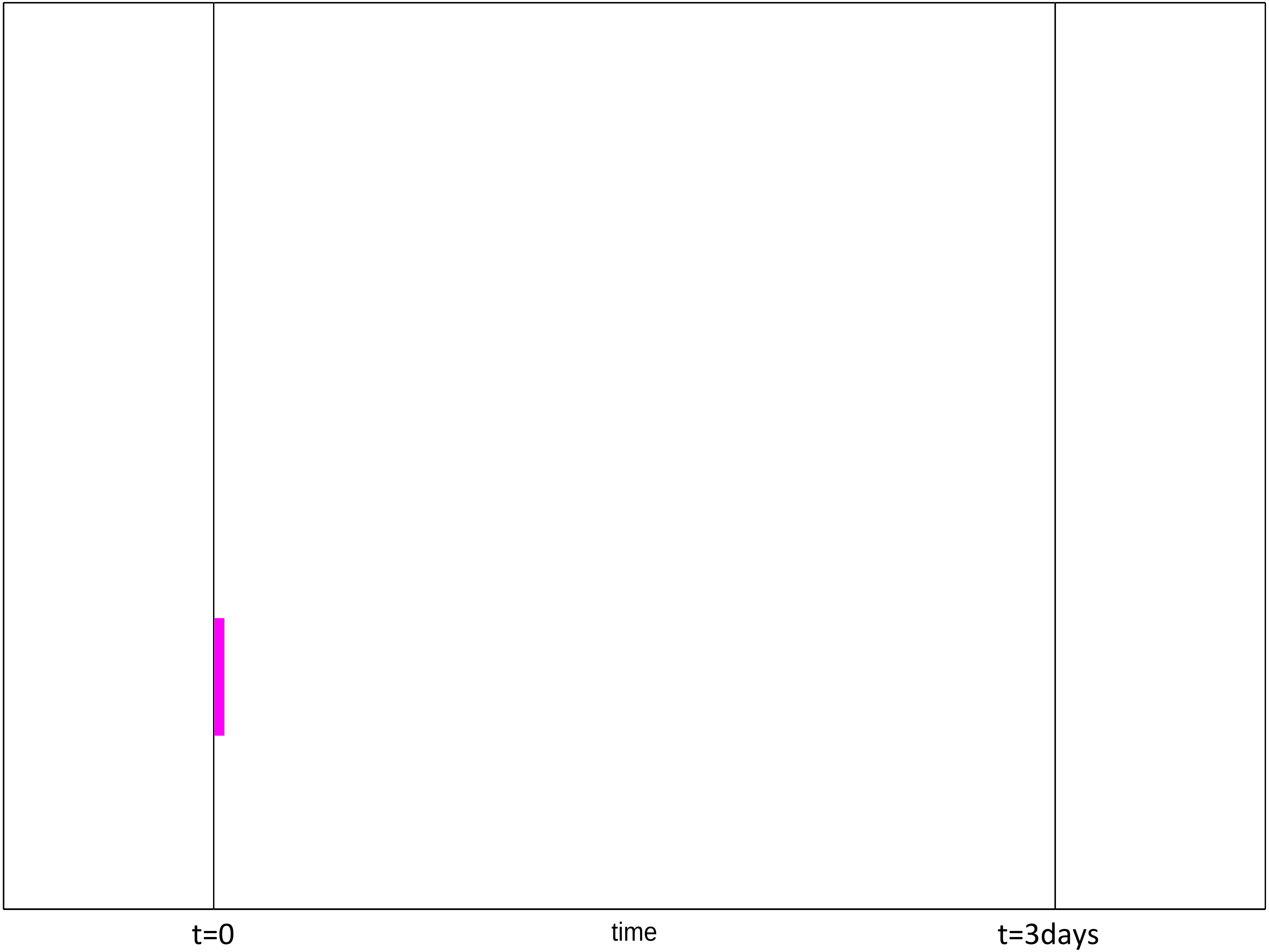
- Can be numerically estimated using linear theory.
- Singular values/vectors are dependent upon the choice of norm; they are critically state dependent (but that's a good thing).

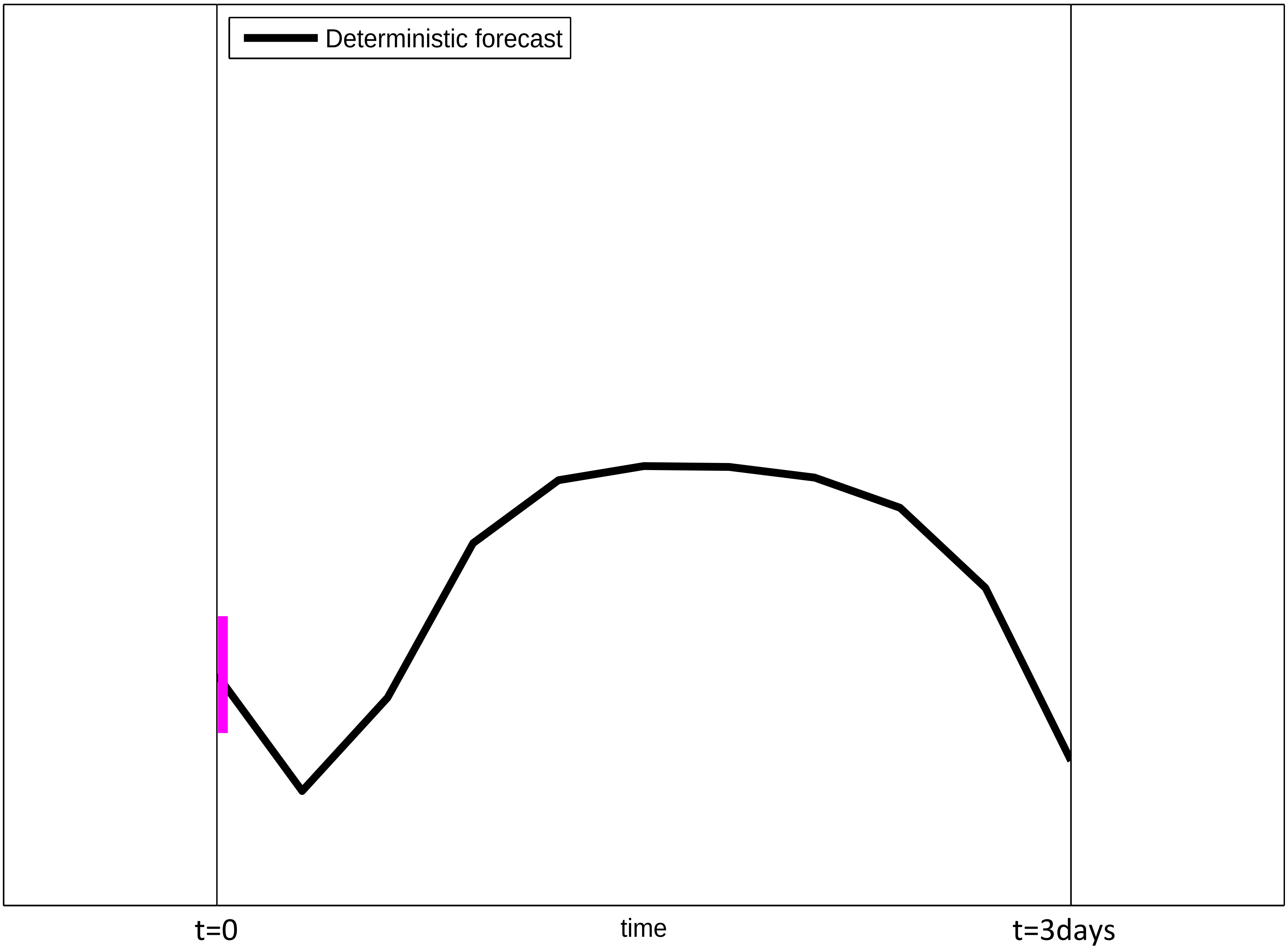




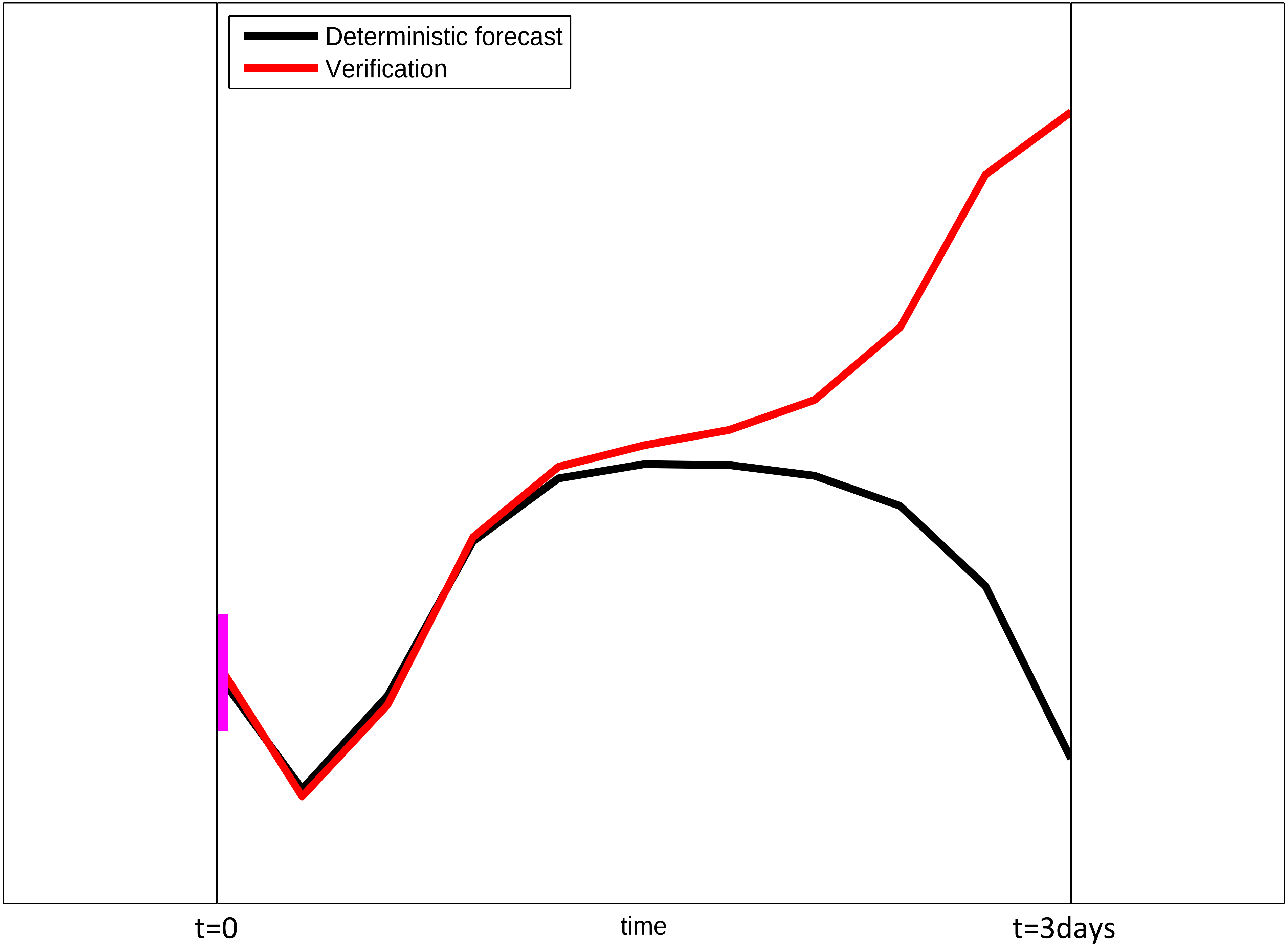


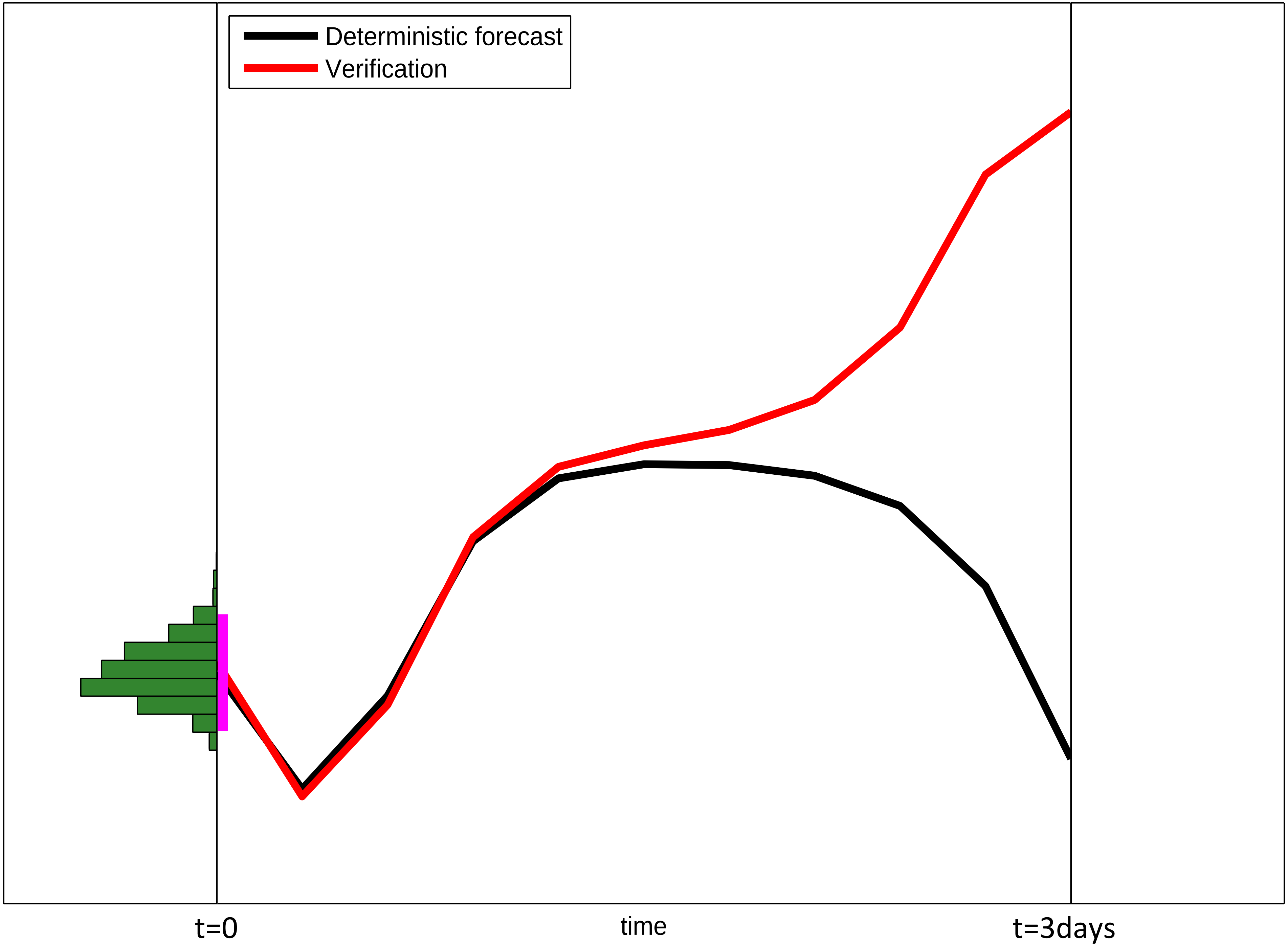


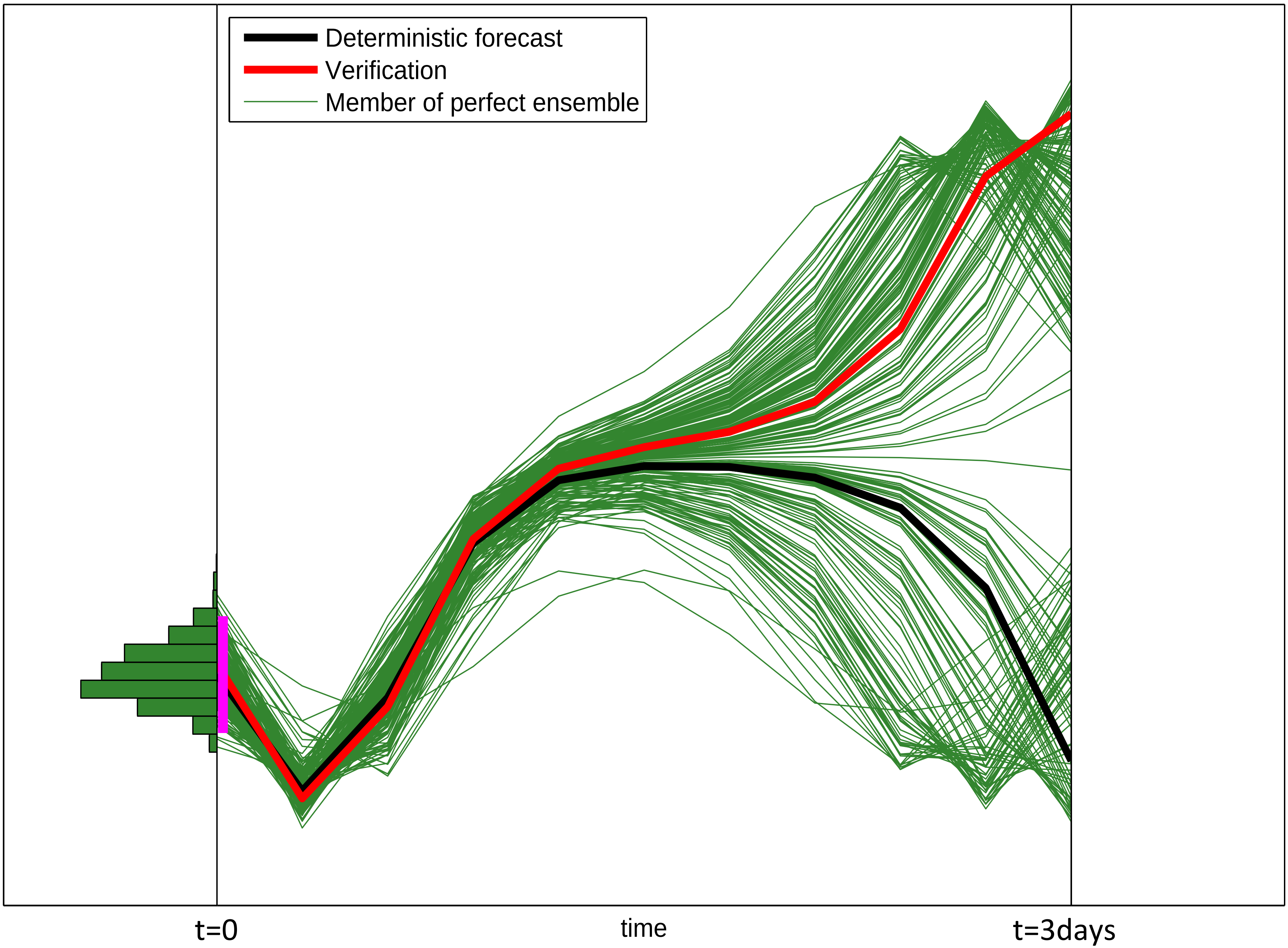

t=0
time
t=3days



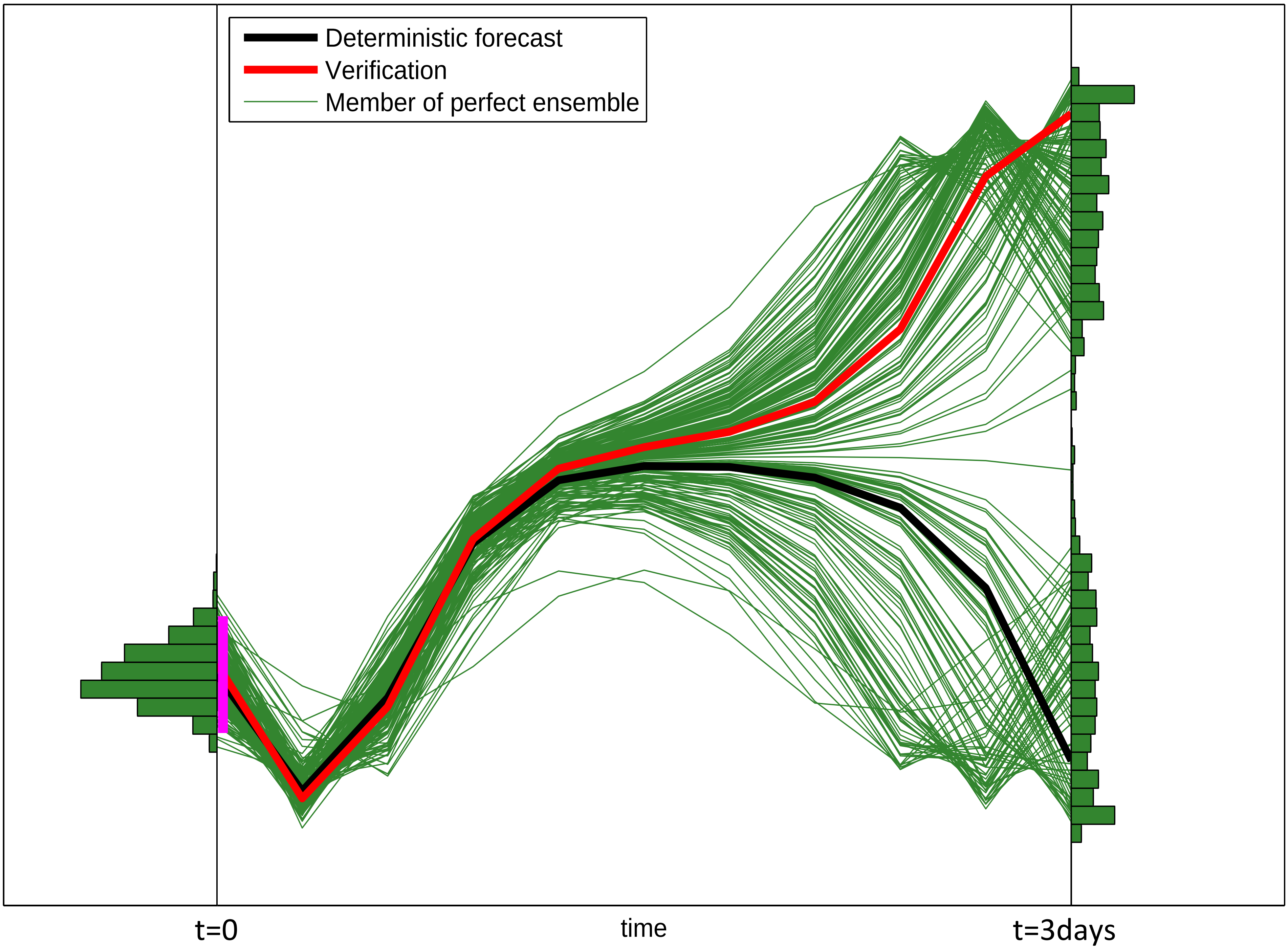
Deterministic forecast
t=0
time
t=3days


Deterministic forecast
Verification
t=0
time
t=3days


Deterministic forecast
Verification
t=0
time
t=3days



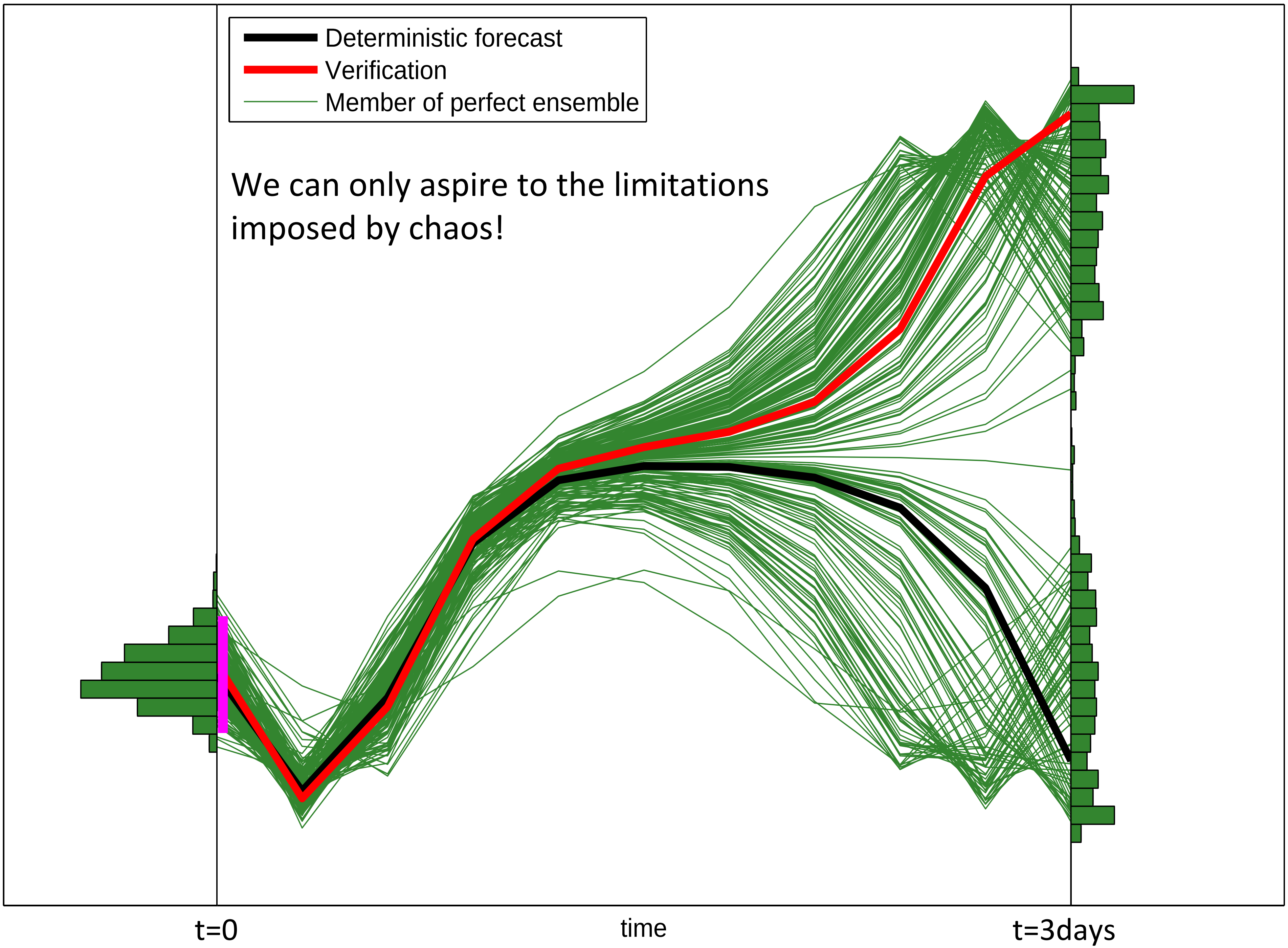
Deterministic forecast
Verification
Member of perfect ensemble
t=0
time
t=3days


Deterministic forecast
Verification
Member of perfect ensemble
t=0
time
t=3days

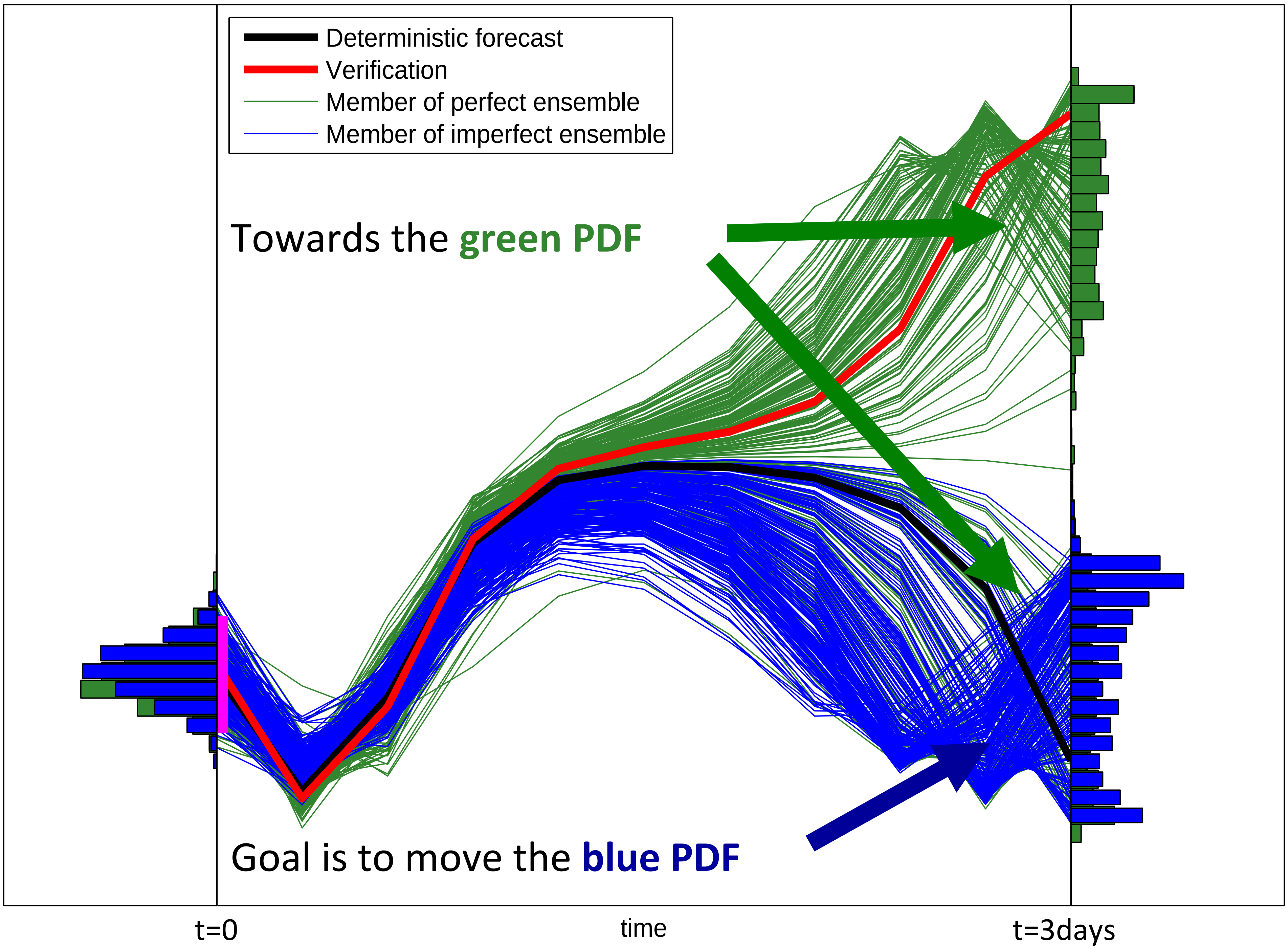
Conclusions I

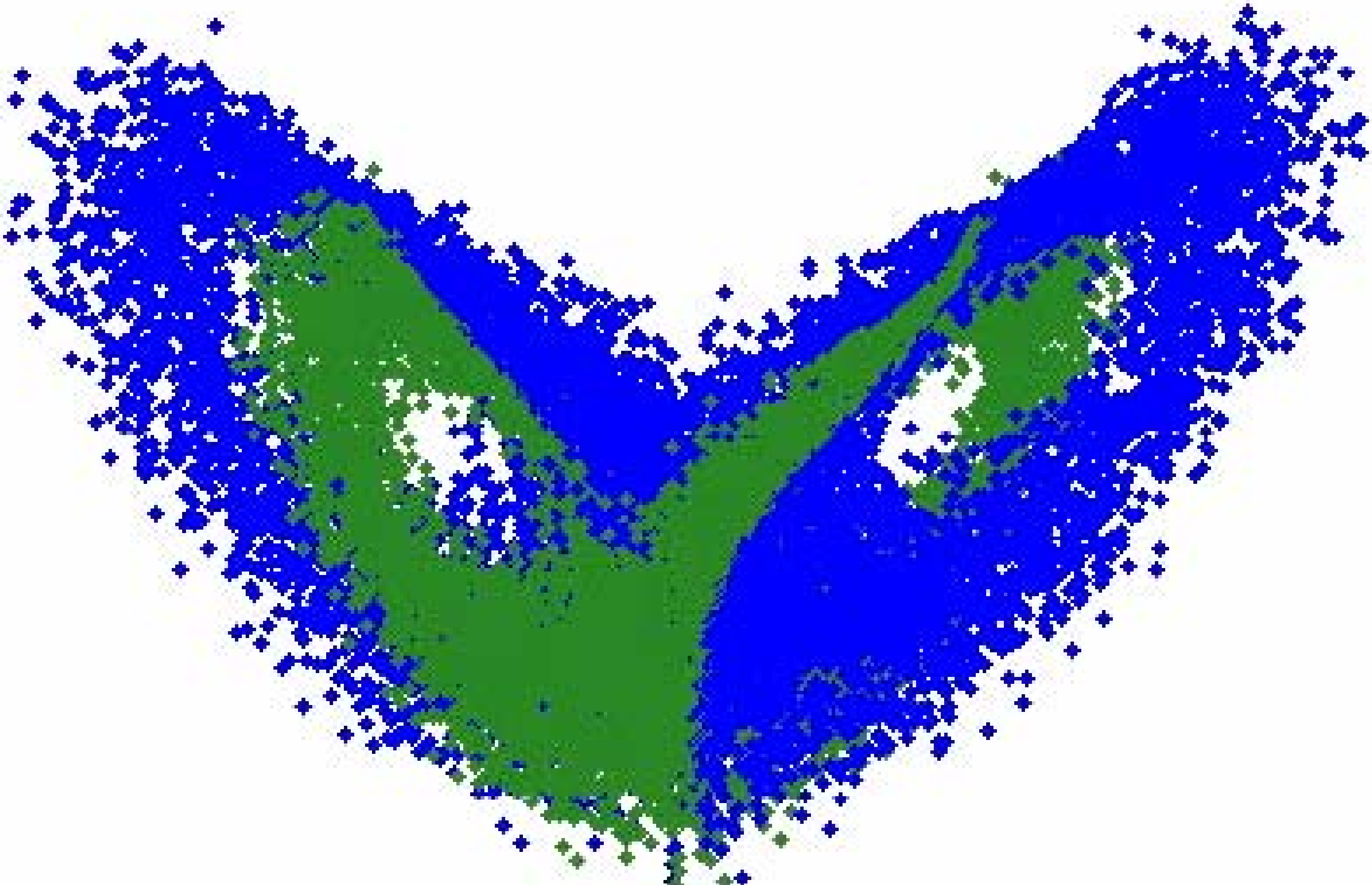
- Chaos is not a problem once we free ourselves from the chains of determinism
  - Chaos is quantifiable
  - Chaos is accountable
- We can use this to our advantage in data assimilation
  - Predict a pdf from a set of initial conditions
  - Consistent with the atmosphere's future pdf if the model is perfect and analysis error is properly estimated in the initial ensemble (both huge caveats)

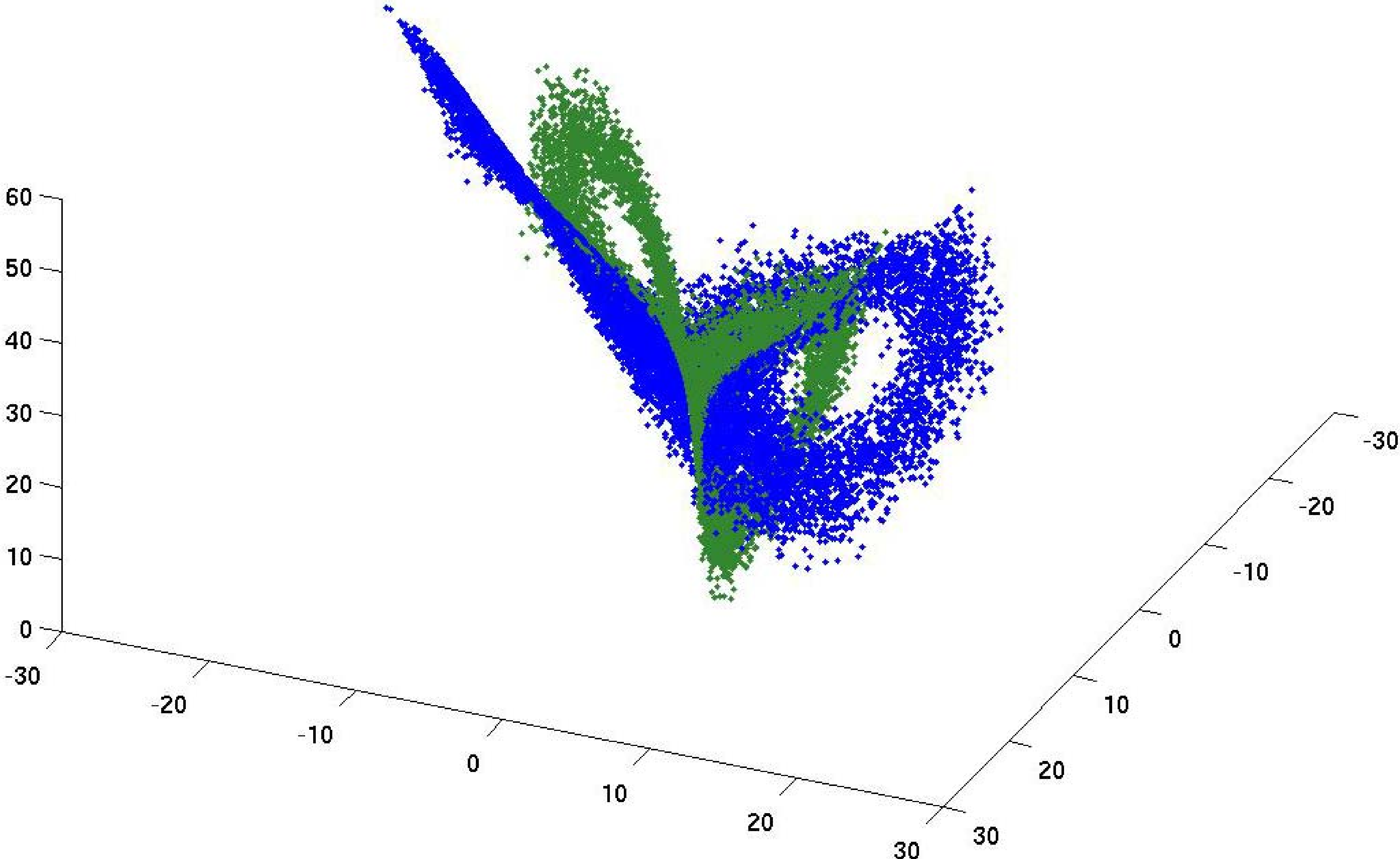
The impact of model inadequacy



Deterministic forecast
Verification
Member of perfect ensemble
We can only aspire to the limitations imposed by chaos!
t=0
time
t=3days


Deterministic forecast
Verification
Member of perfect ensemble
Member of imperfect ensemble
Towards the green PDF
Goal is to move the blue PDF
t=0
time
t=3days

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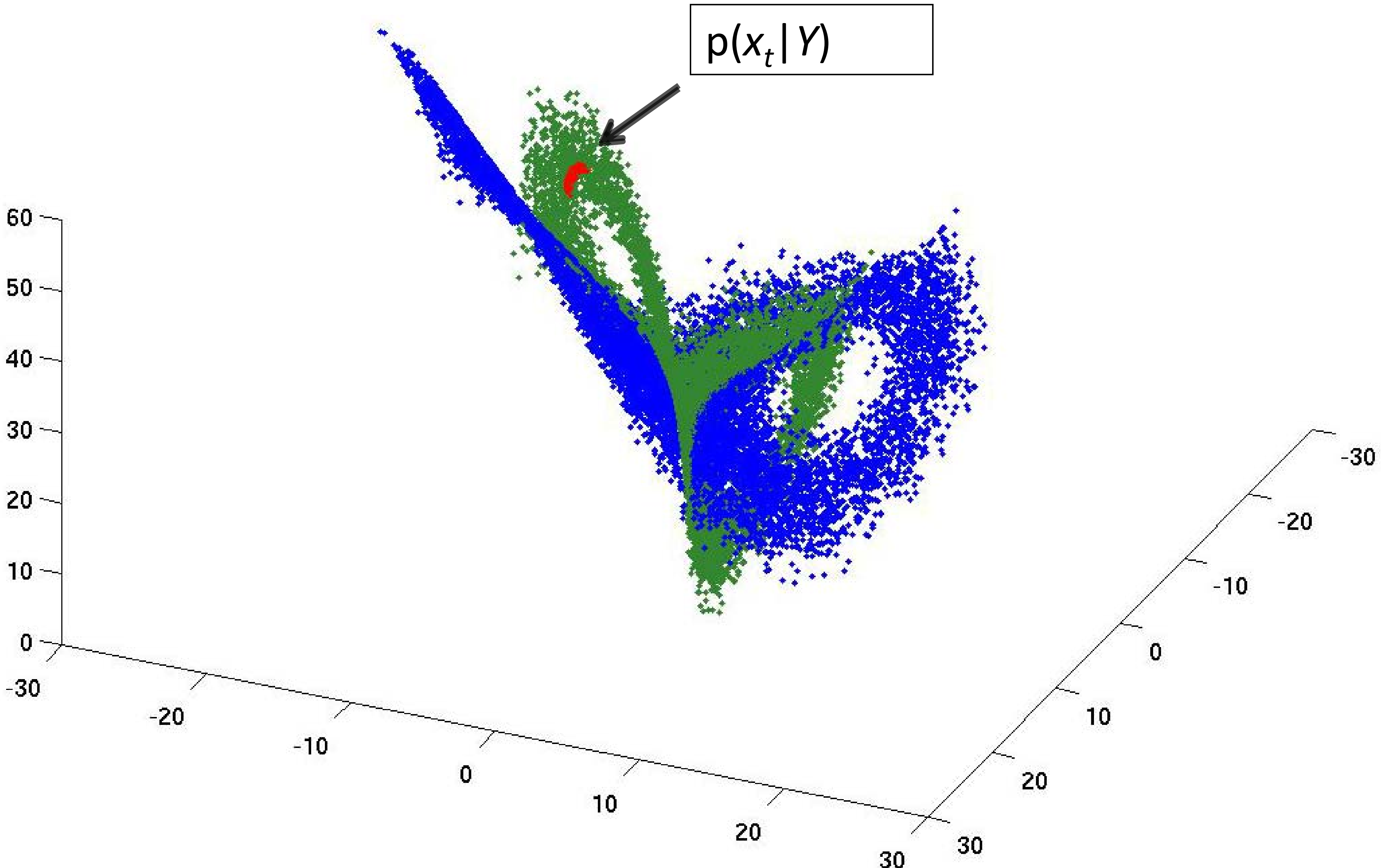

$p(x_t|Y)$
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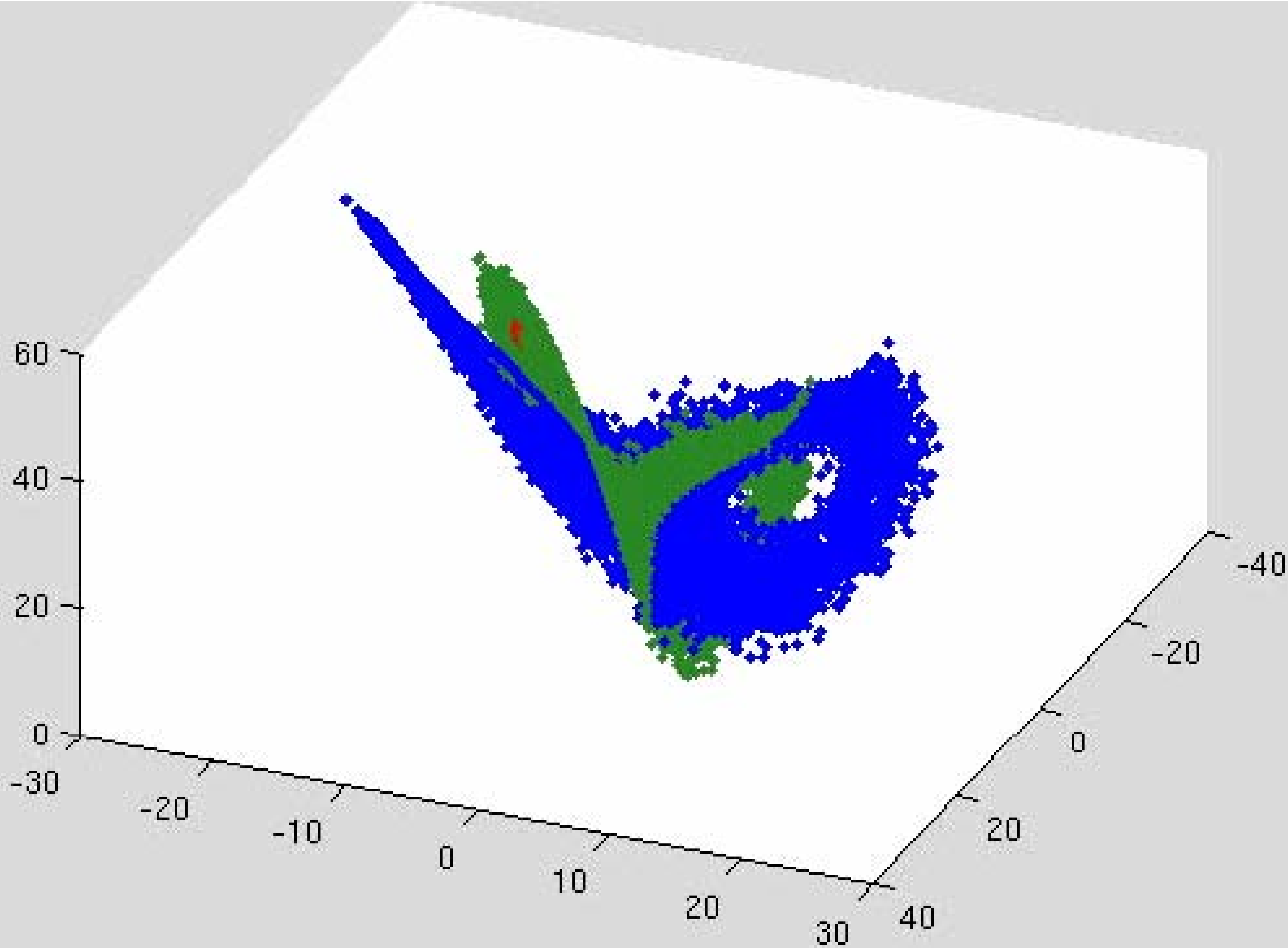
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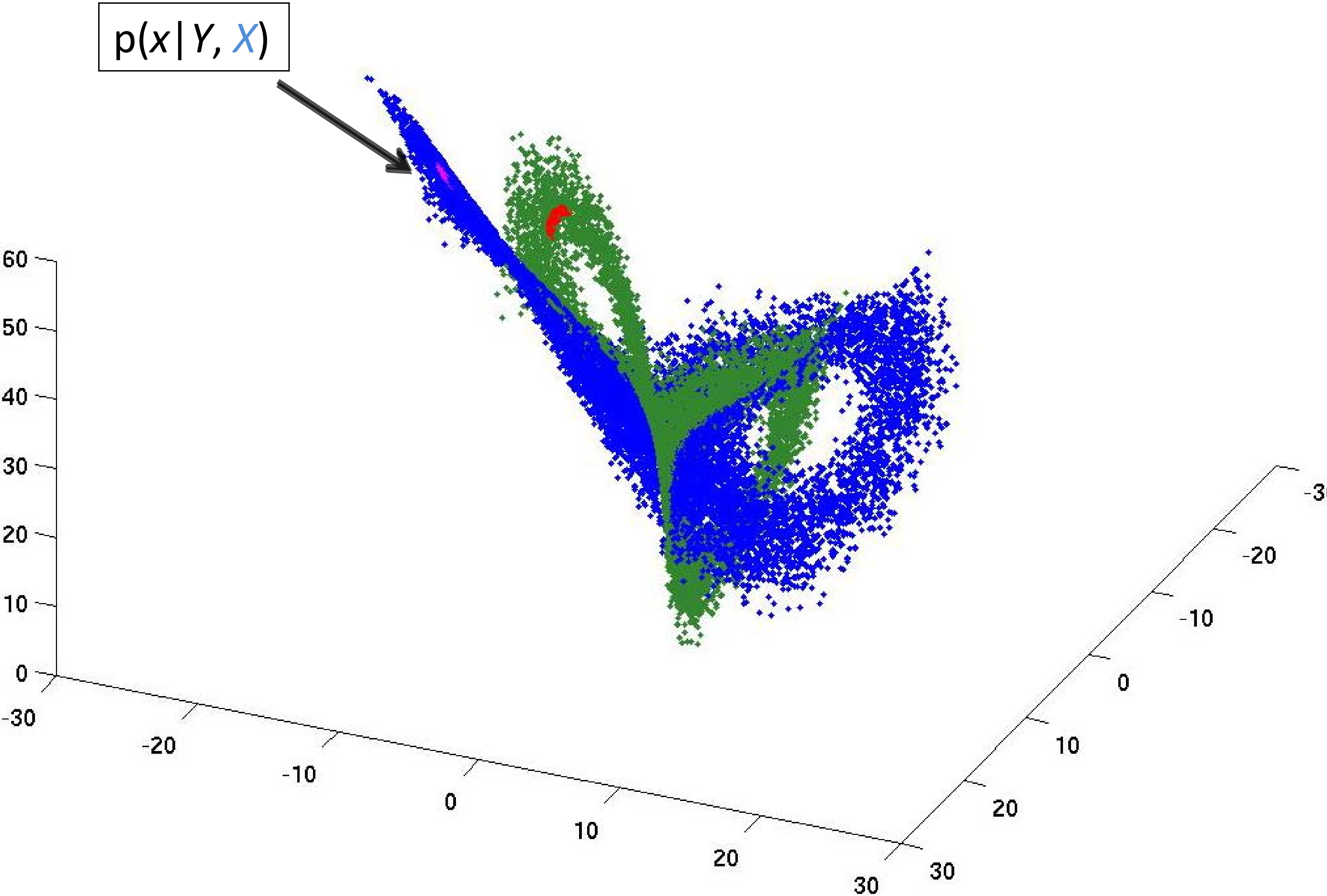

p(x|Y, X)
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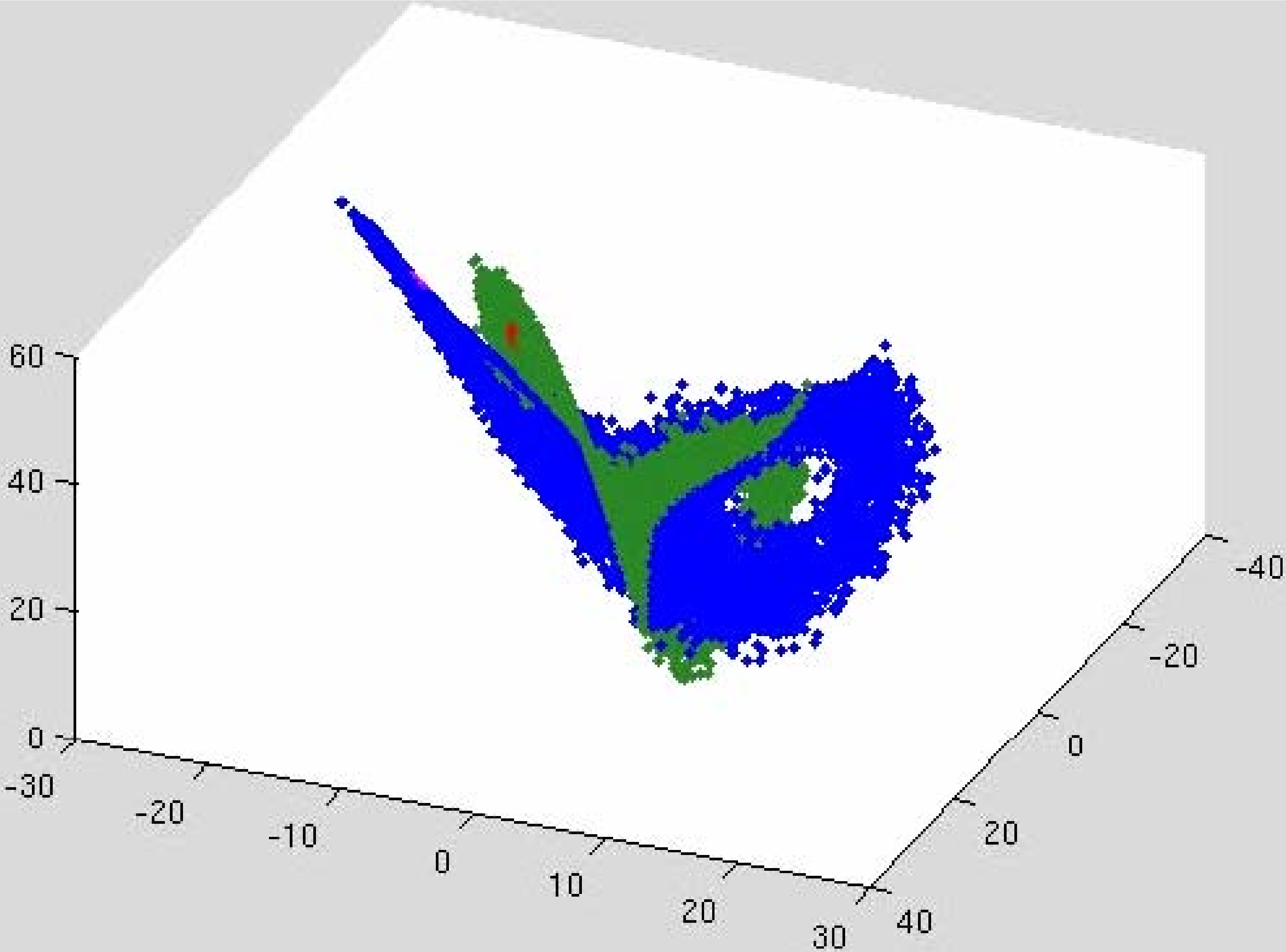

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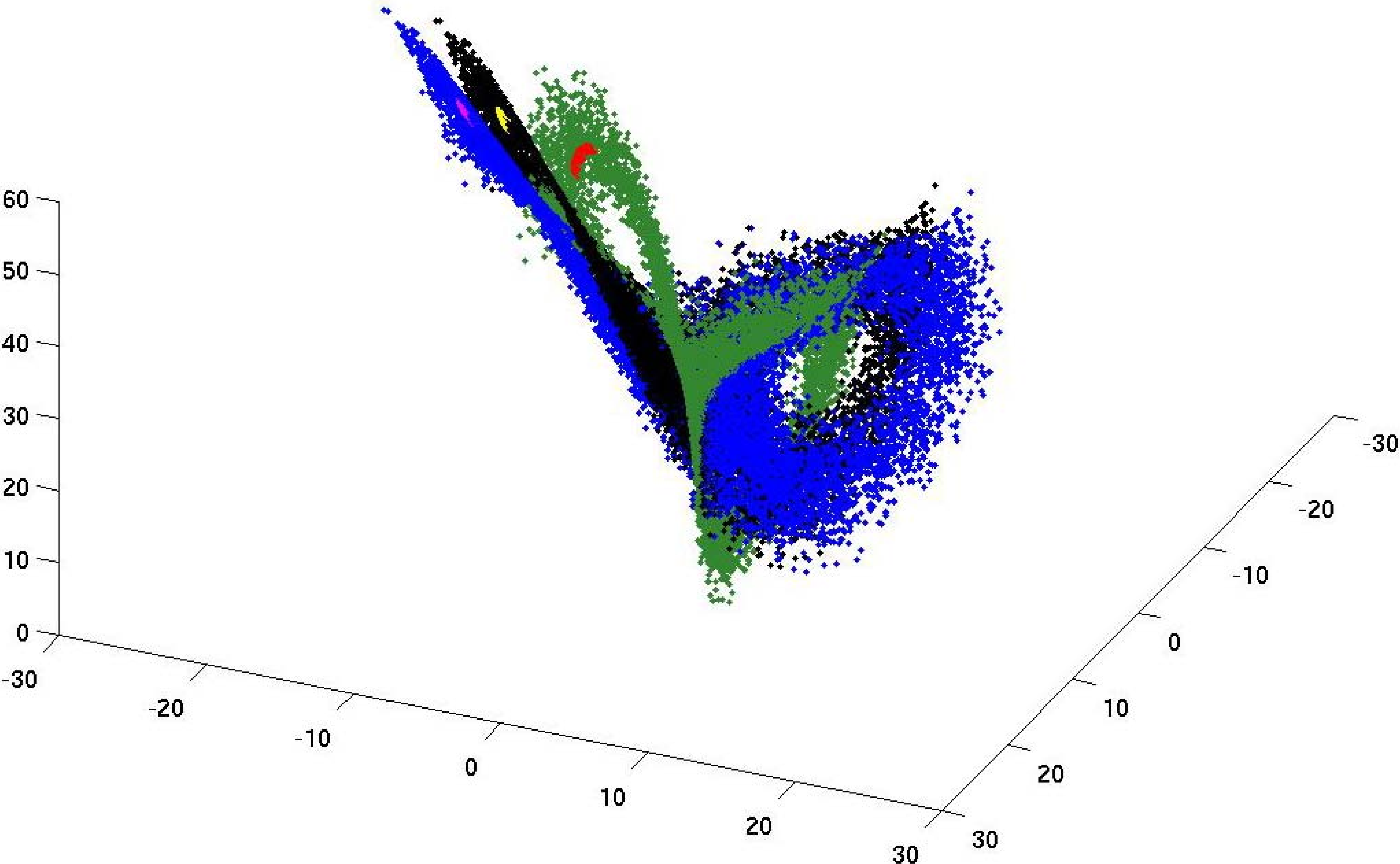
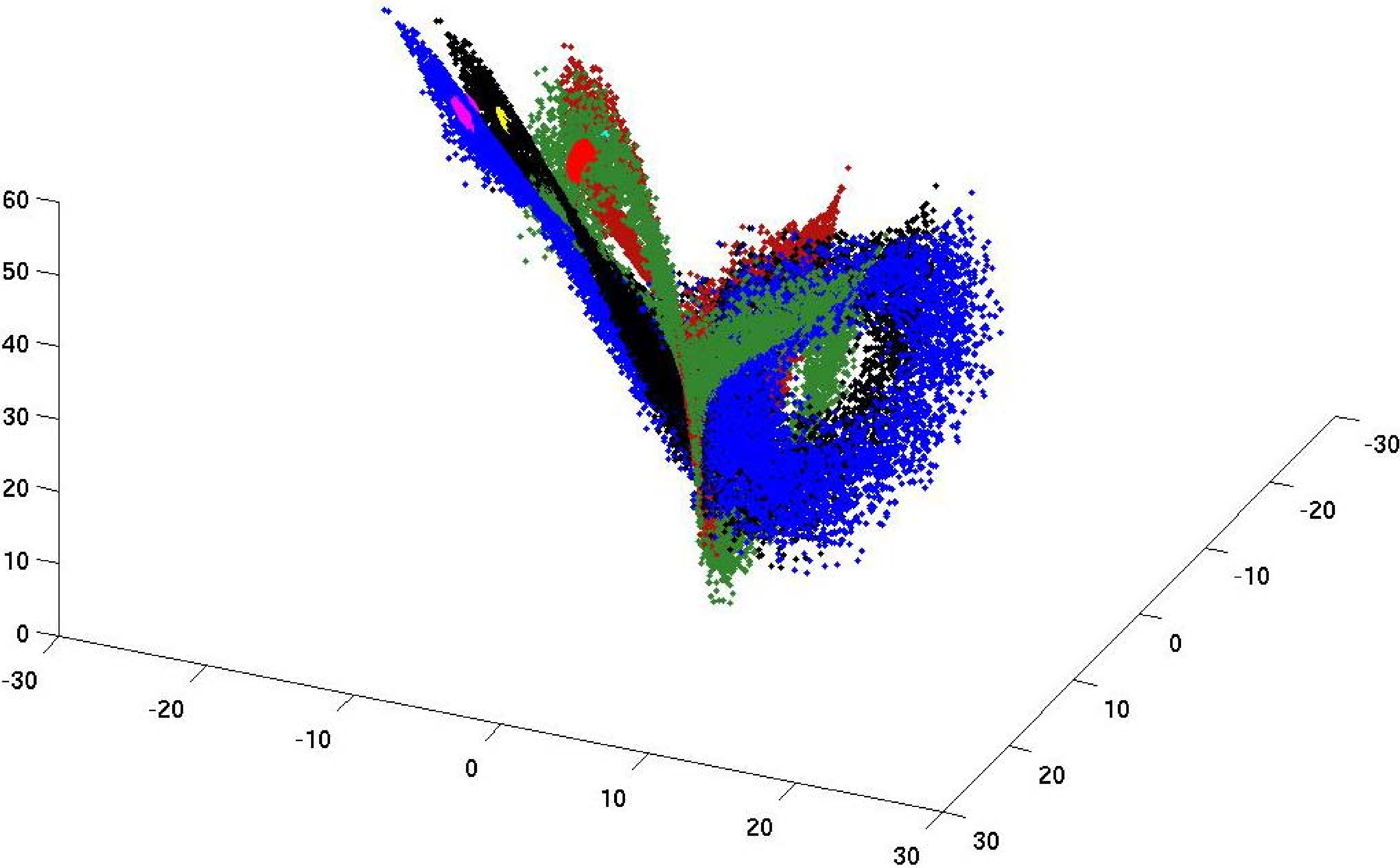
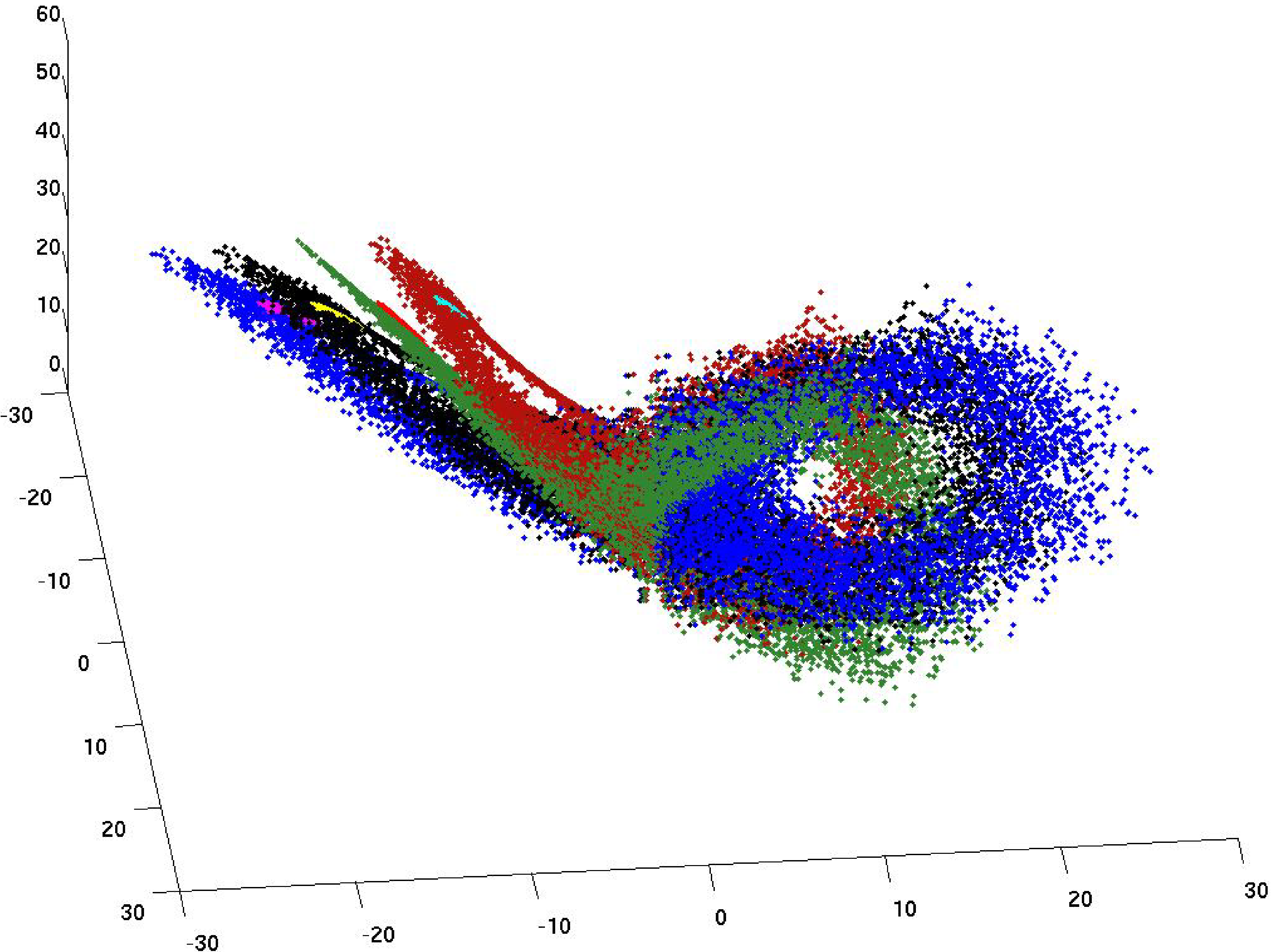
Even though we have p(x|Y, X) for a given model, we cannot get a perfect forecast.



Multi-model ensemble: bounds truth but not a draw from truth





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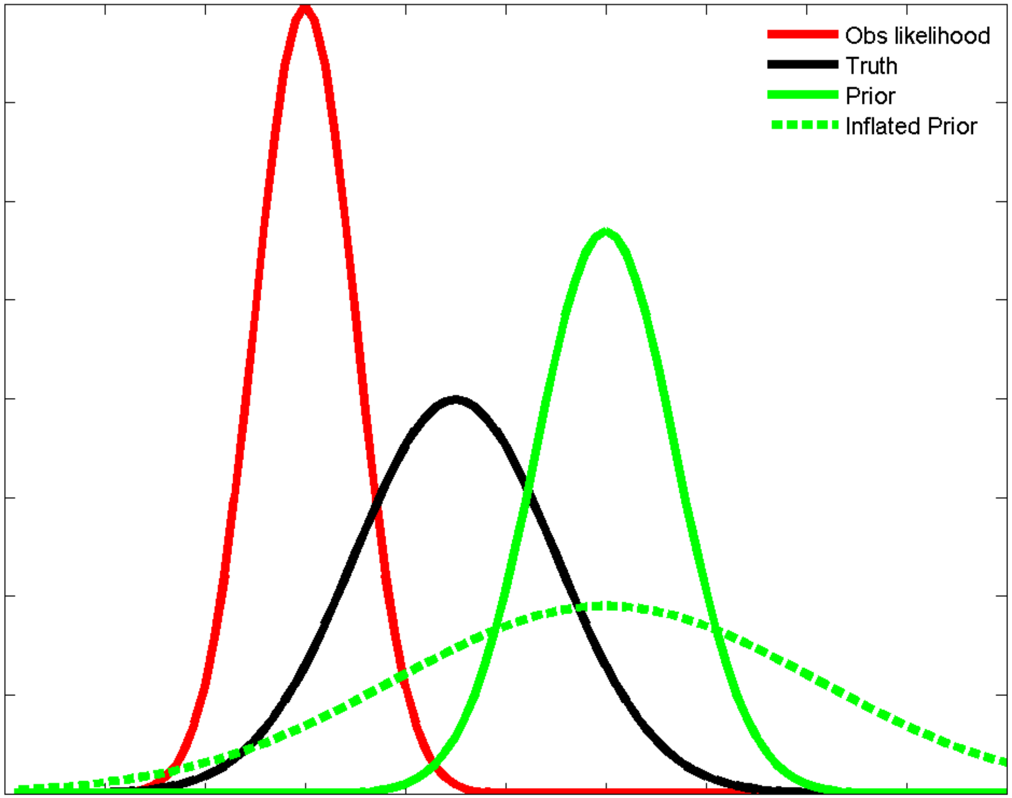
Conclusions II

- Chaos is relatively manageable (we can only aspire to the limitations imposed by chaos); let’s make probabilistic predictions.
- In the same way that initial condition uncertainty guarantees we will never have perfect deterministic forecasts, model uncertainty guarantees we will never have perfect probabilistic forecasts.
- In the same way that deterministic forecasts in the face of initial condition uncertainty are still useful, so too are “probabilistic” forecasts in the face of model uncertainty.

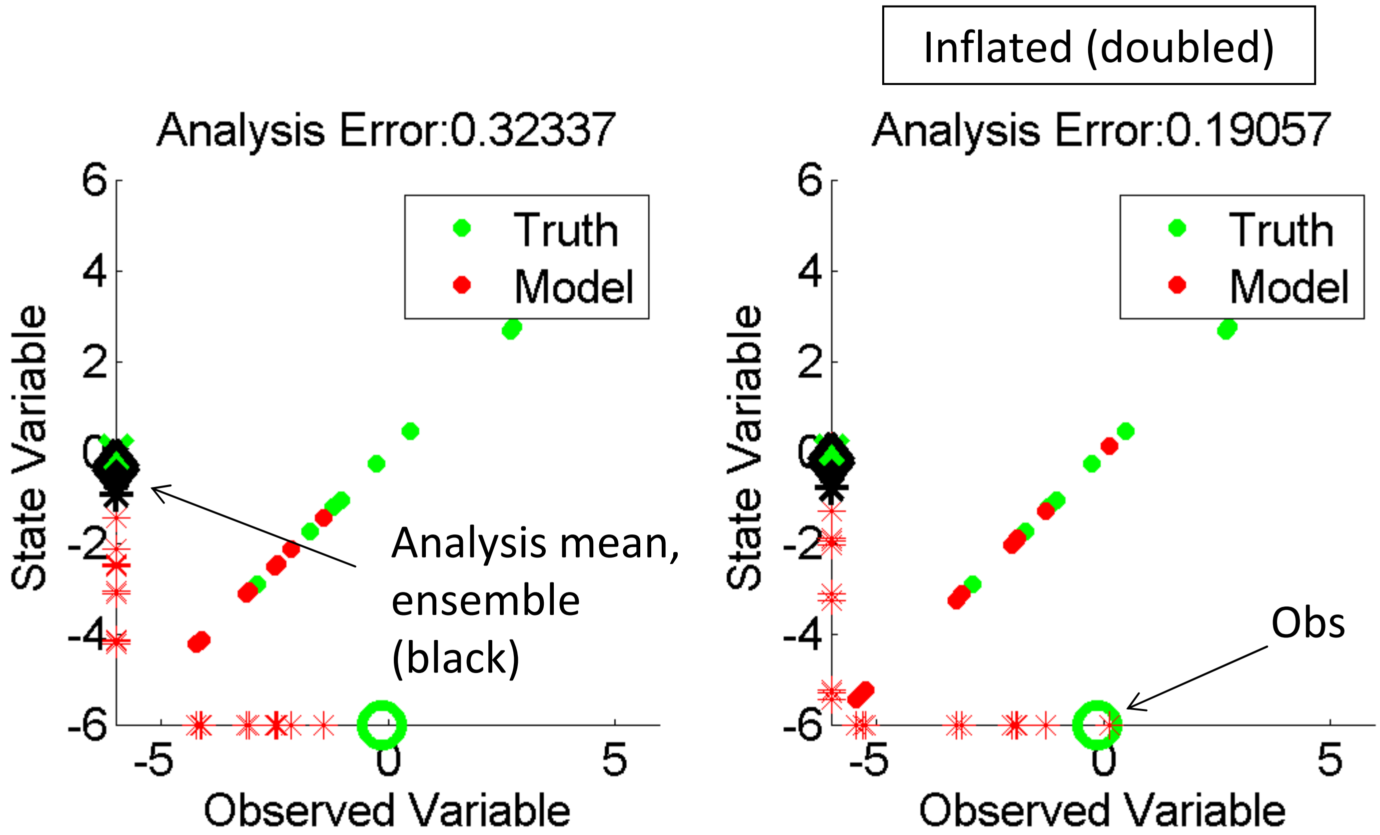
Naïve handling of model error in ensemble filters

- Inflate the prior (background) ensemble by a constant fact before solving the analysis equation.



Model prior has incorrect mean and spread – systematic model errors

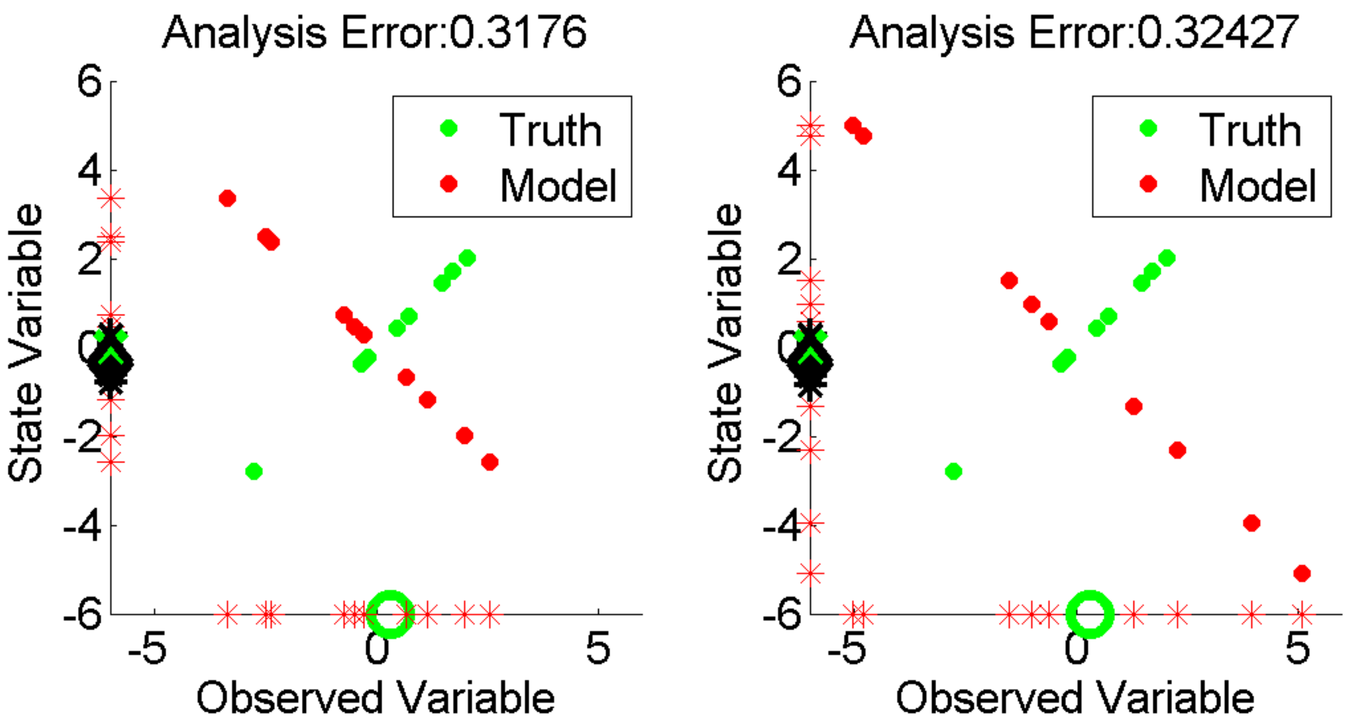
Example 1: inflation works



- Model lies in same plane as truth
- Biased
- Under-dispersive ensemble

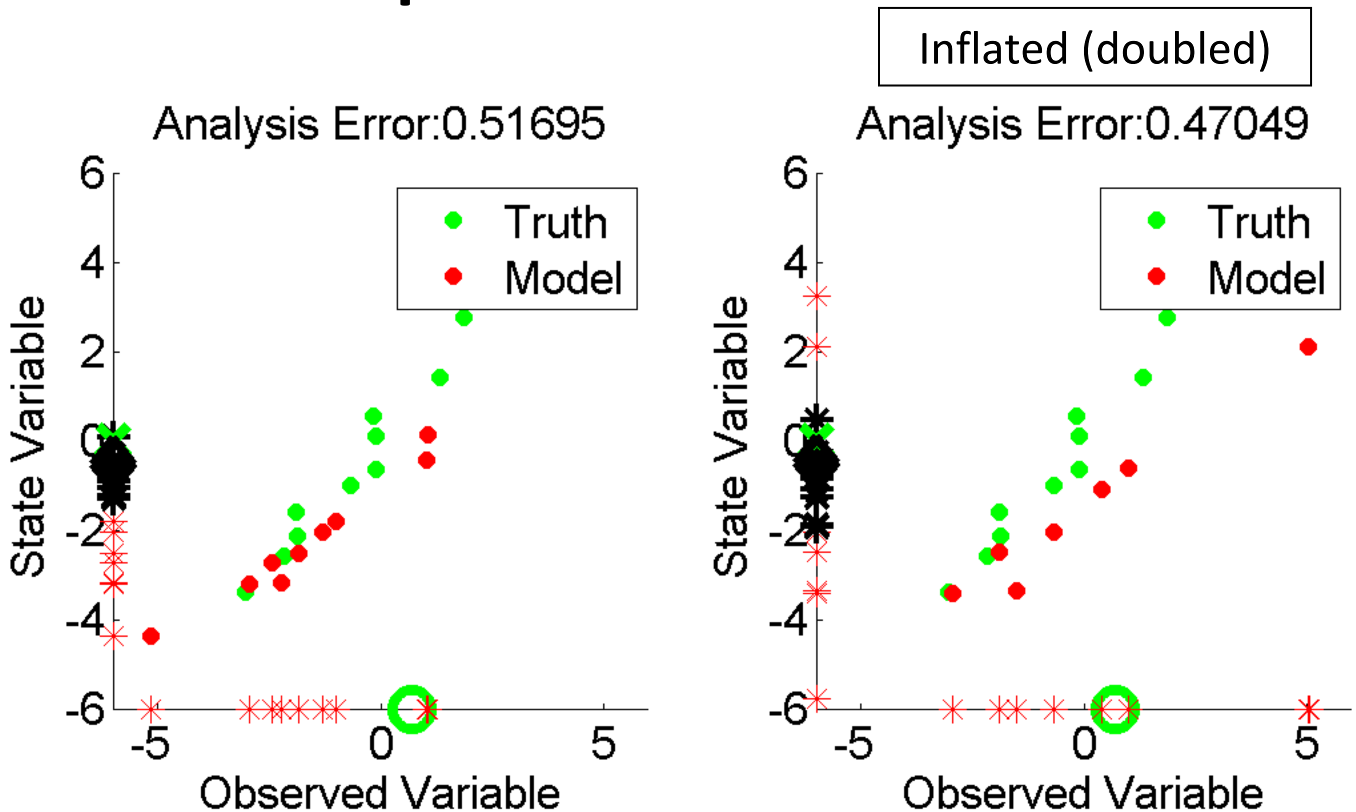
Example 2: doesn't work

Inflated (doubled)



- Model lies on orthogonal plane
- Note orthogonal and biased is worse

Example 3: more realistic



- Imperfect correlations
- Model not on same plane but has a non-zero projection on truth
- In recursive filters we benefit from repeated applications of inflation

Lorenz-96 Example

- A 40-variable model intended to simulate propagating waves around a latitude circle
- Dynamics are spatially invariant (covariances too)
- Forced with a constant on the right-hand side – usually $F=8$ to produce a chaotic system.
- Imperfect model generated with $F\neq8$.

Lorenz-96 Example

- DART_section2.pdf page 36 provides some information on the exercise
  - Launch Matlab
  - Driver script is run_lorenz_96

Thanks to Jeff Anderson and the DART team for the example and documentation materials!

Suggestions for Exploration

- Spin up an ensemble to get a climatological distribution.
- Turn on assimilation and run freely. Does the filter without localization or inflation track the observations?
- Play with localization and inflation separately to see the effects. Put them together.
- Change F to 6 to assimilate with an imperfect model. What happens? What if you don't use inflation and/or localization?